ASYNCHRONOUS MACHINE

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1. Asynchronous machine

The asynchronous machine is the electrical machine most widely used as a motor, it is extensively used in several industry applications and in the rail transport, for instance. However, this machine has also a large use as a generator in electric power plants, particularly in the renewables.

1.1 Typical Layouts

As usual, the design of this rotating machine has two main components, the rotor and the stator. The rotor is the mobile component of the machine and it is located in its inner side. The stator is the fixed component and it is the outside of the machine. Figure 1 shows the stator component. It is built in sheets of ferromagnetic material and, in its inner part, the conductors of the winding are distributed inside slots. This distribution of the conductors is adequate to make a winding with three phases, which is a winding where all the phases are equal but having a spatial phase shift equal to 120º.

Figure 1 – Stator of an asynchronous machine.

There are two main possible rotor designs that are presented in Figure 2. In the left side of the figure it is represented the squirrel cage rotor, a simple and cheap solution that does not enable a galvanic access to the rotor conductors. The other solution, represented in the right side of the figure, is a wound rotor, which has a similar design to the stator’s, but has the conductors at its outer side, and inside slots. The three phase winding of the rotor may be accessed through the rings and brushes also represented in the figure. This design is more expensive than the squirrel cage design; however it has a large application as generator in renewable energy power plants, for instance the wind power plants and the power plants of oscillating wave column (Porto

Cachorro – Azores). The stator of the generator may be connected directly to the mains and an electronic fractional power converter is enough to supply the rotor circuits and to command and smooth the power flow to the mains.

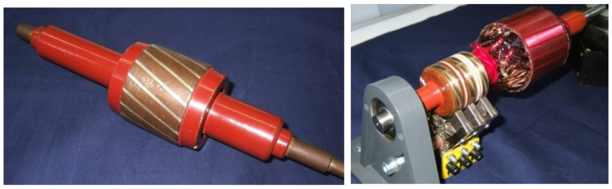

Figure 2 – Different solutions for the rotor of the asynchronous machine.

1.2 Model of a Steady State Regime

It is possible to represent an asynchronous machine in normal operation and in a steady state regime by two sets of three phase circuits as those shown in the diagram of Figure 3. One of the sets of circuits is in the stator and the other is in the rotor. This representation is used for both design solutions of the rotor.

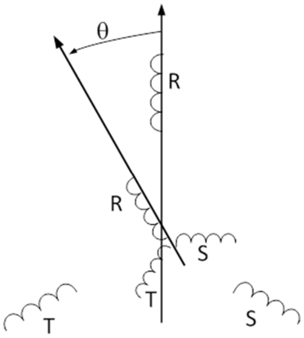

Figure 3 – Circuit diagram of a three phase asynchronous machine.

In a steady state regime the currents in the circuits of the machine are sinusoidal waveforms and are generically represented by expressions (1)

$$\begin{aligned} i_{1e} &= I_e \cos(\omega t) & i_{1r} &= I_r \cos(\omega_r t + \xi) \\ i_{2e} &= I_e \cos(\omega t - 2\pi/3) & i_{2r} &= I_r \cos(\omega_r t + \xi - 2\pi/3) \\ i_{3e} &= I_e \cos(\omega t - 4\pi/3) & i_{3r} &= I_r \cos(\omega_r t + \xi - 4\pi/3) \end{aligned} \tag{1}$$

1. Problem

Consider a three phase sinusoidal distributed winding with *p* pole pairs.

a) Find the angular speed of the rotating magnetic field generated by this winding when the phase conductors carry a three phase system of currents like those represented by relations (1).

b) Repeat precedent question, but consider that phase *2* carries current i_3 and phase 3 carries current i_2.

Hints: The radial component of the magnetic field distribution generated by phase *1* may be represented by the relation $B = Acos(p \propto)i_1$.

When in the circuits of the stator flows a three phase current, a sinusoidal rotating induction magnetic field distribution is generated in the air gap of the machine. This distribution rotates with angular speed equal to ω / p, where *p* is the number of pole pairs of the machine.

2. Problem

An asynchronous machine with two pole pairs has stator and rotor windings like the ones described in precedent problem. The rotor rotates with a speed equal to 1420 rpm and the frequency of the power supply is equal to 50Hz. Find the frequency of the induced currents on rotor windings when the stator winding carries a three phase balanced system of currents as those referred in items of previous problem.

Hints: Note that the direction of the magnetic rotating field is different for the two cases.

If the rotor rotates with angular speed $\dot{\theta}$ the speed of the wire conductors of the rotor relatively to the induction magnetic field distribution is equal to

$$\frac{\omega}{p} - \dot{\theta} \tag{2}$$

As the machine has *p* pole pairs, each winding of the rotor develops an *emf* which frequency may be calculated by equation (3) that also assumes the form (4) when the fractional slip parameter is used (5).

$$\omega_r = \omega - p\dot{\theta} \tag{3}$$

$$s\omega = \omega - p\dot{\theta} \tag{4}$$

$$s = \frac{\omega - p\dot{\theta}}{\omega} \tag{5}$$

3. Problem

Determine the rotational relative speed of the magnetic field distribution generated by the induced currents you determined in precedent problem. Consider two reference frames: one tied to the rotor and the other tied to the stator.

The currents in the rotor windings also have the frequency given by (4). This three phase current system also contributes for the induction magnetic field distribution in the air gap of the machine. Indeed, the sinusoidal waveform of the induction magnetic field distribution rotates with frequency $s\omega / p$ relatively to the rotor. This is a similar situation to that verified at the stator. The speed of this distribution relatively to the stator is equal to its speed relatively to the rotor plus the speed of the rotor as in (6).

$$\frac{s\omega}{p} + \dot{\theta} = \frac{\omega}{p} \tag{6}$$

In a similar way, we can conclude that in the conductors of the stator windings, there are *emfs* with frequency ω due to this distribution.

The situation that was described may be equivalent to the one where the air gap's magnetic induction field distribution is generated by the currents that flow in two different magnetically coupled coils with the same number of turns, one representing the stator and the other representing the rotor windings. This way, the current and the linked flux in each coil may be represented, respectively for the stator and for the rotor, by phasors (7) and (8).

$$\overline{I}_e \, e^{j\omega t} \qquad \overline{\phi}_M \, e^{j\omega t} \tag{7}$$

$$\overline{I}_r \, e^{js\omega t} \qquad \overline{\phi}_M \, e^{js\omega t} \tag{8}$$

The mutual flux depends on a contribution of both currents and it is possible to write relation (9).

$$\begin{aligned} \overline{\phi}_M e^{j\omega t} &= M\overline{I}_m e^{j\omega t} = M\,(\overline{I}_e e^{j\omega t} + \overline{I}_R e^{j(s\omega t + \dot{\theta} t)}) \\ \overline{\phi}_M e^{js\omega t} &= M\overline{I}_m e^{js\omega t} = M\,(\overline{I}_e e^{j(\omega t - \dot{\theta} t)} + \overline{I}_R e^{js\omega t}) \end{aligned} \tag{9}$$

Using this representation and considering resistance and leakage effects it is possible to write the equations (10) for the two equivalent coils. These equations may also take the form given by (11).

$$\begin{cases} \overline{U}_e e^{j\omega t} = r_e \overline{I}_e e^{j\omega t} + j\omega l_e \overline{I}_e e^{j\omega t} + j\omega M \overline{I}_m e^{j\omega t} \\ \overline{U}_R e^{js\omega t} = r_R \overline{I}_R e^{js\omega t} + js\omega l_R \overline{I}_R e^{js\omega t} + js\omega M \overline{I}_m e^{js\omega t} \end{cases} \tag{10}$$

$$\begin{cases} \overline{U}_e = r_e \overline{I}_e + j\omega l_e \overline{I}_e + j\omega M \overline{I}_m \\ \dfrac{\overline{U}_R}{s} = \dfrac{r_R}{s} \overline{I}_R + j\omega l_R \overline{I}_R + j\omega M \overline{I}_m \end{cases} \tag{11}$$

Figure 4 presents an equivalent circuit of an asynchronous machine in steady state regime. This means that the equations of this circuit are equations (11). In this example we consider a short circuit rotor operation, as is the case of a squirrel cage asynchronous machine.

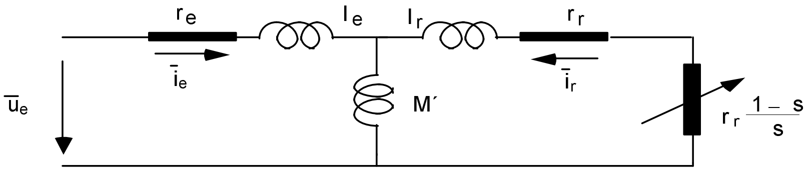

Figure 4 – Equivalent single phase circuit representation of an asynchronous machine in steady state regime.

In order to simplify the first approach analyses it is common to use a simplified model of an asynchronous machine like the one shown in Figure 5. This equivalent circuit representation corresponds to an ideal machine without magnetic leakages, without resistance in the stator windings, very high magnetic permeability of ferromagnetic materials and null air gap.

4. Problem

Consider the circuit shown on Figure 4 4 and using KVL and KCL verify that equations (11) represent also the behavior of this circuit when rotor circuits are short-circuited.

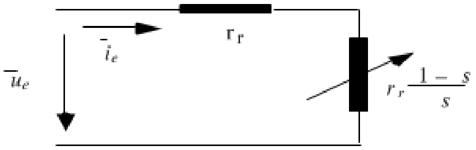

Figure 5 – Equivalent circuit representation of an ideal asynchronous machine.

1.3 Modes of Operation of the Asynchronous Machine

It is possible to use the equivalent circuit representation to obtain some important characteristics of the asynchronous machine, as for instance the torque speed characteristic, represented in Figure 6, for a four poles machine.

In the equivalent circuit representation note that a part of the electric power received by the stator

$$P_e = 3U_e I_e \cos(\varphi) \tag{12}$$

Is dissipated in the stator and rotor windings

$$p_e = 3r_e I_e^2 \qquad p_R = 3r_r I_r^2 \tag{13}$$

The rest[1] of the electric power is converted into mechanical power and it is quantified by the power "dissipated" in the virtual and variable resistance of the equivalent circuit.

$$P_m = 3r_r \frac{1-s}{s} I_r^2 \tag{14}$$

Using this value we can determine the torque developed by the machine (16). To obtain this value it is enough to divide the mechanical power (14) by the value of the rotor speed (15).

$$\dot{\theta} = \frac{\omega(1-s)}{p} \tag{15}$$

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[1] In a real machine there are other losses, like the losses in the magnetic circuit that can be quantified using a resistor in parallel with magnetization coil.

$$T_e = 3 p r_r \frac{1}{s\omega} I_r^2 \tag{16}$$

For each value of the speed – fractional slip -, knowing the applied voltage to the stator circuits and its frequency, the rotor current can be calculated solving the equivalent circuit. Using this value in equation (16), the value of the torque is calculated. Repeating this procedure, it is possible to find a torque / speed characteristic like the one presented in Figure 6.

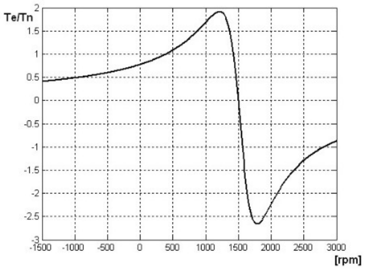

Figure 6 – Torque speed characteristic of a two pole pairs asynchronous machine.

The normal operation zone of the asynchronous machine is near the synchronous speed where the torque is equal to zero. In the example of Figure 6the synchronous speed is equal to 1500 rpm. This normal zone of operation is characterized by small slip value and the torque speed characteristic may be approached by a linear function. If we know the rated value of the torque T_N and its rated speed N_N, we can approach the torque / speed characteristic by the expression (17), where N_S is the synchronous speed.

$$T = \frac{T_N}{N_N - N_S}(N - N_s) \tag{17}$$

Note that when the speed is higher than the synchronous speed the asynchronous machine operates as an electric generator. It receives mechanical power from the rotor and supplies electric power through the stator circuits.

5. Problem

A three phases, 2 pole pairs asynchronous machine has the following rated values:

$$U = 400V \quad f = 50Hz \quad I = 8A \quad cos(\varphi) = 0.8$$

$$P_m = 3.5kW \qquad N = 1420rpm$$

a) Find the efficiency of the motor at rated regime.

b) Determine the speed when the motor drive a torque load whose value is equal to an half rated torque.

Hints: Note that torque is null at synchronous speed. Use a linear approach for the torque speed characteristic.

1.4 Variable Speed Operation

In many applications the rotor speed must change. Under these circumstances the power supply frequency must also change to maintain a small slip to ensure low copper losses. Besides, it is necessary to change the stator voltage to ensure the same magnetization conditions of the magnetic circuit. In a first approach these objectives may be attained maintaining a constant relation between the voltage and the frequency in the stator circuits. These objectives are implemented by the use of the adequate power electronics converters.

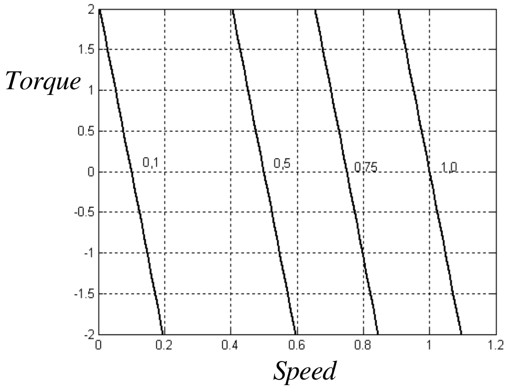

Figure 7 – Normalized torque speed characteristic of an asynchronous machine for different supply frequencies.

6. Problem

A three phases, 3 pole pairs asynchronous machine with a V/f mode operation, has the following rated values:

$$U = 400V \quad f = 50Hz$$
$$P_m = 250kW \quad s_N = 8\%$$

a) Determine the voltage applied to the stator circuits and the synchronous speed when the frequency is 25Hz.

b) Determine mechanical power received by the machine if it works with 25Hz and with a slip equal to -8%.

Hints: The V/f mode operation maintains a linear relation for torque speed characteristic at different frequency.

Note that the rate of the electronic power converters must be equal to the rate of the asynchronous machine, because all the power converted goes through the stator circuits and through the power electronic converters. These connections to the mains using the power electronic converters are a drawback of this system. So, in some applications, namely in renewable electric power plants, for instance wind and oscillating wave column power plants, it is used a DFIG solution- double feed induction generator. In these cases the asynchronous machine must be of wound rotor type.

The main feature of this system is to use the supply through the rotor circuits to control and command the machine in variable speed operation.

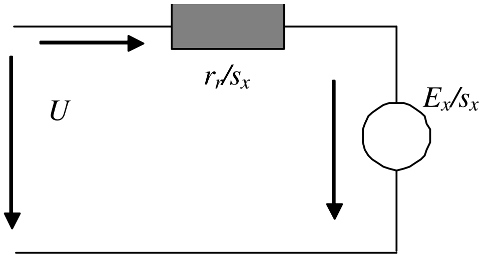

Figure 8 – An ideal circuit representation of a DFIG machine.

The command of the machine is made using electronic power converters connected to the rotor circuits. These equipment change the electromotive force E_x in Figure 8 to control the rotor current and power flow through the rotor circuits. This process only needs about 1/3 of the rated power of the asynchronous machine. Therefore the power electronics converters may be designed for this low power value. This is one of the main advantages of these systems.

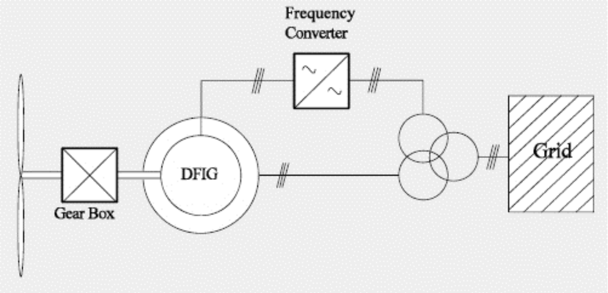

Figure 9 – Main components of a wind DFIG system.

Figure 9 shows the main components of a wind DFIG generator. Observe the direct connection of the stator circuits of the machine to the mains.

7. Problem

A 50 Hz DFIG carries a current *I* with speed of 1580rpm and with the circuits on the rotor short-circuited.

a) Find the speed if the machine carries the same current but the circuits on the rotor have an applied voltage equal to 20% of the voltage on the stator.

b) Plot the torque speed characteristic for different rotor voltages.

Hints: Use the simplified circuit representation of DFIG.